SOLUTION QUANTITATIVE TOOLS IN MANAGEMENT MAY 2010

QUESTION 1

a)

Annual Gross Income	Mid-point (x)	Population (f)	fx	fx^2
Less than 6000	5000 ( )	60	300000	1500000000
6000 and less than 8000	7000	260	1820000	12740000000
8000 and less than 10000	9000	310	2790000	25110000000
10000 and less than 14000	12000	220	264000	31680000000
14000 and less than 20000	17000	20	2040000	34680000000
20000 and less than 32000	26000	10	520000	13520000000
32000 and above	38000 ( )		380000	14440000000
		1000	10490000	133670000000

The mean: $x = \frac{\sum fx}{\sum f}$

$= \frac{10490000}{1000}$ ( )

$= 10490$ ( )

The Mode: $\text{Mode} = L + \frac{D_1 i}{D_1 + D_2}$

Where $L = 8000$

$D_1 = 310 - 260 = 50$

$D_2 = 310 - 220 = 90$

$i = 200$

$M_o = 8000 + \frac{50 \times 2000}{50 + 90}$

$= 8714.29$

The Medium $M_e = L + \frac{\left(\frac{\sum f}{2} - \sum fbm\right)}{Fm} i$

Where $L = 8000$

$\sum f = 1000$

$\sum fbm = 320$

$fm = 310$

$i = 2000$

$\therefore M_e = 8000 + \frac{\left(\frac{1000}{2} - 320\right)}{310} \times 2000$

$= 9161.29$

(ii) Since (mean > medium > model), the distribution of annual gross incomes in this suburb is positively skewed (i.e. skewed to the right).

(b) The coefficient of variation is given as:

$$CV = \frac{\text{Standard deviation}}{\text{Mean}} \times 100\%$$

where standard deviation = s = $\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$

$$= \sqrt{\frac{13367000000}{1000} - \left(\frac{10490000}{1000}\right)^2}$$

= 4861.06 ( )

and mean = x = 10490

$$CV = \frac{4861.06}{10490} \times 100\% \quad (\ )$$

= 46.3% ( )

(c) The appropriate curve to show the distribution of taxes is the Lorenz curve, as shown below (see graph attached)

Taxes paid	Percentage Cum Taxes	Population	Percentage Cum Pop
6000	1.5	60	6
20000	6.4	260	32
66000	22.8	310	63
70000	40.1	220	85
74000	58.4	120	7
68000	75.2	20	99
100000	100	10	100

QUESTION 2

(a) (i) What quantity of stock to order at any one time.

(ii) How frequently to order

(iii) Stock holding quantity

(iv) Time to place an order

(v) Take advantage of discount offers.

(b) Costs involved in stock control are:

- Order costs – Order costs are costs incurred whenever a stock order is generated. There might involve the costs relating to clerical, administrative and managerial activities linked to the order process, costs of transportation, costs of receiving and inspecting orders, costs of finance and accounting support.
- Purchase cost – Purchase cost is the actual cost of purchasing the items from the suppliers.
- Holding costs – Holding costs are those associated with the company holding a fixed quantity of stock over a given period of time. Holding costs can include the cost of capital tied up in the value of the stock, storage costs, (cooling, lighting, security), depreciation, insurance and obsolescence.
- Stockout costs – These are costs incurred when stock is not available. Stockout cost may appear in the form of lost of goodwill or higher prices from another supplier or simply lost profits.

(c) Annual Demand : $D = 1550$ packets

Number of working days/year: $= 310$ days

Set-up cost : $C_s = GH¢300$

Holding cost : $C_h = GH¢360$/packet/year

Daily demand : $d = \frac{1550}{310} = 5$ packets/day

Daily production : $p = \frac{7750}{310} = 25$ packets/day

(i) Optimum production lost size is

$$EBQ = \sqrt{\frac{2CsD}{\left(1 - \frac{d}{p}\right) C_h}}$$

$$= \sqrt{\frac{2 \times 3000 \times 1550}{(1 - 5/25) \times 360}}$$

$$= 179.7 = 180 \text{ packets}$$

(ii) Maximum Inventory $= \left(1 - \frac{d}{p}\right) EBQ$

$$= (1 - 5/25) \times 180$$

$$= \underline{\underline{144}} \text{ packets}$$

(iii) Number of production runs $= \frac{D}{EBQ}$

$= \frac{1550}{180}$

$=$ 8.6 times/year

$=$ approximately 9 times/year

(iv) Production run time:

$t_1 = \frac{EBQ}{p}$

$t_1 = \frac{180}{25}$

$=$ 7.2 days

(v) Production cycle time:

$t = \frac{EBQ}{d}$

$= \frac{180}{5}$

$=$ 36 days

:. Time between production runs is:

$t_2 = t - t1$

$= 36.0 - 7.2$

$=$ 28.80 days

(vi) Annual inventory cost:

$$TC = \frac{DCs}{EBQ} + \left(\frac{d}{1-p}\right)\frac{EBQ}{2}Ch$$

$$= \frac{1550 \text{ x } 3000}{180} + \left(\frac{5}{1-25}\right) \text{ x } \frac{180}{2} \text{ x } 360$$

$=$ GH¢51,753.33

SOLUTION QUANTITATIVE TOOLS IN MANAGEMENT MAY 2010

QUESTION 3

(a) Let x represents quantity of Doclean (in litres)

y represents quantity of Maclean (in litres)

Then the problem can be formulated as follows:

Max Z = 25x + 18y

s.t. $30x + 48y \leq 480$ (Labour in stage I)

$30x + 75y \geq 600$ (Labour in stage II)

$5x + 20y \leq 180$ (Mix A)

$10x + 30y \geq 240$ (Mix B)

$X, y \geq 0$ (non-negativity)

(b) $30x + 48y = 480$ ---------- L_1

when x = o, y = 10 (0, 10)

y = o, x = 16 (16, 0)

$30x + 75y = 600$ ----------L_2

when x = o, y = 8 (0, 8)

y = o, x = 20 (20, 0)

$5x + 20y = 180$ ----------L3

when x = o, y = 9 (0, 9)

y = o, x = 36 (36, 0)

$10x + 30y = 240$ -----------L4

when x = o, y = 8 (0, 8)

y = o, x = 24 (24, 0)

See graph sheet attached for graph.

The feasible region is bounded by ABCD.

Extreme Point	Z = 25x + 18y
A (0, 9)	162
B (2.667, 8.333)	216.669
C (6.857, 5.714)	274.277
D (0, 8)	144

Hence the optimum production-mix is 6.857 litres of Doclean and 5.714 litres of Maclean.

(c) Labour in stage I is a binding constraint
Mix B is also a binding constraint
Labour in stage II and Mix A are non-binding constraint.

(d) (i)
$$30x + 48y = 481 \text{ ------------ } (1)$$
$$10x + 30y = 240 \text{ ------------ } (2)$$

$$(2) \times 3 \equiv 30x + 90y = 720 \text{ ------------ } (3)$$

$$(3) - (1) \equiv 42y = 239$$
$$y = 5.690$$

$$\therefore 10x + 30(5.69047619) = 240$$
$$x = \mathbf{6.}928571429$$

$$Z_{new} = 25(6.928571429) + 18(5.69047619)$$
$$= 2753.6428571$$
$$= 275.64$$

Hence the shadow price for labour in stage I constraint is: 275.64 – 274.28 = GH¢1.36/minute

(ii) ie For every 1 minute used after the 480 minutes of the labour in stage I, profit should increase by GH¢1.36.

QUESTION 4

(a) Next year's matrices are

$$N = \begin{pmatrix} 11,280 & 624 & 252 \\ 14,400 & 780 & 384 \\ 17,040 & 864 & 552 \end{pmatrix}$$

$$M = \begin{pmatrix} 11,990 & 715 & 275 \\ 15,180 & 792 & 418 \\ 18,150 & 957 & 594 \end{pmatrix}$$

$$P = M - N$$

$$= \begin{pmatrix} 710 & 91 & 23 \\ 780 & 12 & 34 \\ 1110 & 93 & 42 \end{pmatrix}$$

(b) Let x people buy GH¢4.00 denomination
and y people buy GH¢8.00 denomination

(i) For required return of GH¢ (in thousand)

$x + y = 10000$

$4 + dy = k$

$$\begin{pmatrix} 1 & 1 \\ 4 & 8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1000 \\ k \end{pmatrix}$$

(ii) We use augmented matrix to obtain the increase of $\begin{pmatrix} 1 & 1 \\ 4 & 8 \end{pmatrix}$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 4 & 8 & 0 & 1 \end{array}\right) \quad \left(\begin{array}{cc|cc} 1 & 0 & 2 & ¼ \\ 0 & 1 & 1 & ¼ \end{array}\right)$$

Therefore

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & -¼ \\ -1 & ¼ \end{pmatrix} \begin{pmatrix} 10000 \\ 56000 \end{pmatrix}$$

or $$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20{,}000 - \frac{56{,}000}{4} \\ -10{,}000 + \frac{56{,}000}{4} \end{pmatrix}$$

(iii) From …. part,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 20{,}000 - \frac{56{,}000}{4} \\ -10{,}000 + \frac{56{,}000}{4} \end{pmatrix}$$

Thus, 6000 tickets of GH¢4.00 denomination and 4,000 tickets of GH¢8.00 denomination will be sold.

QUESTION 5

a) Let the probability of event A be Pr (A)
P (A) denotes the numerical measure of the likelihood of occurrence of event A

$0 \leq Pr\ (A) \leq 1$

Pr (A) = 0, the event A is impossible to occur
Pr (A) 1, at event a is certain to occur
Pr (Ậ) = 1 – Pr (A)
Pr (A) = 0.5, the event A is just as likely to occur or not.

b) (i) Blue die

1	(1, 1, T 1, 1, H)	(1, 2, T 1, 2, H)	(1, 3, T 1, 3, H)	(1, 4, T 1, 4, H)	(1, 5, T 1, 5, H)	(1, 6, T 1, 6, H)
2	(2, 1, T 2, 1 H)	(2, 2, T 2, 2, H)	(2, 3, T 2, 3,H)	(2, 4, T 2, 4, H)		
3						
4						
5						
6						

(ii) Pr (total score 8, H) $= \frac{5}{72}$

(iii) Pr (total score 8) $= \frac{10}{72} = \frac{5}{36}$

(iv) Pr (odd number score less than 7 and a tail) $= \frac{6}{72} = \frac{1}{12}$

c) i. Mutually exclusive events

Two or more events which have no common outcomes. If A, B are events that are mutually exclusive, then $A \cap B = \emptyset$ and Pr $(A \cap B) = O$

<u>Ext events</u>

If the sample space $S = A \cup B \cup C$ and A, B, C are the only events

<u>Independent events</u>

Two or more events are independent if the probability of occurrence of one is not influenced by the occurrence or nonoccurrence ie of the other(s).

Let M and E represent the event of a choosing a man and an employed person respectively.

ii. Pr $(M \cap E) = \frac{500}{1100} = \frac{5}{11}$

iii. Pr $(E \cap M) = \frac{200}{1100} = \frac{2}{11}$

iv. Pr $(M \cap E) \cup (M \cap E)) = \frac{100}{1100} + \frac{300}{1100} = \frac{400}{1100}\ \frac{4}{11}$

SOLUTION QUANTITATIVE TOOLS IN MANAGEMENT MAY 2010

QUESTION 6

(a) The coefficient of determination can be interpreted as:

(i) a measure of reliability of an estimate

(ii) the proportion of total variation in the dependent variable as explained by the inclusion of the independent variable(s).

(b) (i) The least squares regression equation is given as:

$$Y = a + b\,x$$

where Y is the profit (in GH¢'000)
X is the sales (in GH¢'000)
a and b are numbers given by:

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$$

$$a = \frac{\sum y - b\sum x}{n}$$

X	Y	XY	X^2	Y^2
748.82	42.13	31547.99	560731.39	1774.94
377.04	24.39	9196.01	142159.16	594.87
166.93	7.77	1297.05	27865.62	60.37
140.78	6.32	889.73	19819.01	39.94
702.11	37.48	27010.17	492958.45	1479.94
41.54	-0.32	-13.29	1725.57	0.10
96.85	3.65	353.50	3979.92	13.32
109.05	4.31	470.01	11891.90	18.58
50.84	-2.69	-136.76	2584.71	7.24
141.57	6.39	904.63	20042.06	40.83
265.28	17.48	4637.09	70373.48	305.55
91.80	7.21	661.88	8427.24	51.98
2932.61	155.11	76817.81	1358578.59	4387.66

$$\therefore\ b = \frac{12 \times 76817.81 - 2932.61 \times 155.11}{12 \times 1358578.59 - 2932.61^2} = 0.0606$$

$$a = \frac{155.11 - 0.0606 \times 2932.61}{12} = 1.8886$$

Hence, $\hat{y} = -1.8886\ |+ 0.0606\,x$

(ii) The regression coefficient is b = 0.0606
ie profits are expected to increase by GH¢60.6 for every GH¢1000 increase in sales.

(iii) (x) when X = 40;

$$\hat{Y} = 1.8886 + 0.006\,(40)$$
$$= 0.5354$$

(ß) when X = 400

$$\hat{Y} = 1.88886 + 0.0606\ (400)$$
$$= 22.3514$$

(iv) The estimate in (x) is not reliable since x = 40 lies outside the range of values of X used in finding the regression equation.

The estimate in (ß) is reliable since X = 400 lies within the range of values of X used in finding the regression equation.

(v) The correlation coefficient (r) is:

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum x^2 - (\sum x)^2]\ [n\sum y^2 - (\sum y)^2]}}$$

$$= \frac{12 \times 76817.81 - 2932.61 \times 155.11}{\sqrt{[12 \times 1358578.59 - 2932.61^2]\ [12 \times 4387.66 - 155.11^2]}}$$

$$= 0.995$$

∴ Coefficient of determination = $r^2 \times 100\%$

$$= 0.995^2 \times 100\%$$

$$= 99\%$$

Hence the estimation in b (iii) are 99% reliable.

QUESTION 7

(a) The expected monetary value (EMV) of a business decision is the average return that can be expected, taking into account probabilities. The EMV is calculated by multiplying the estimated value of the possible outcomes by the associated probabilities and then summing.

The EMV is a useful measure in business as it allows decision-makers to compare alternative decisions. The highest EMV the criterion employed to choose among alternative strategies.

(b) (i) <u>The Decision Tree</u>

(ii) At node a; EMV = 50000 x 0.8 + 70000 x 0.2
= GH¢26000

At node c; EMV = 60000 x 0.7 +-15000 x 0.3
= Gh¢37500

At node b; EMV = 37000 x 0.5 + 20000 x 0.5
= GH¢8750

At node d; EMV = 0 x 0.6 + 15000 x 0.25 + -2000 x 0.15
GH¢3450

Hence, the best course of action is to expand the business by relocating to a new site .

(c) (i) ……… weighing less than 92 kg is ……. from the standard variable.

$$Z = \frac{92 - 95}{8} = 1.67$$

From to …..

Pr ….. weighing less than 9241 = $\underline{0.5}$
-0.4525
0.0475

(ii) Standardising, $z = \frac{97 - 95}{1.8} = 1.11$

:. Pr …… weighing more than 97 kg = -<u>0.3665</u>
……..

2