

**NOVEMBER 2017 PROFESSIONAL EXAMINATIONS  
QUANTITATIVE TOOLS IN BUSINESS (PAPER 1.4)  
CHIEF EXAMINER'S REPORT, QUESTIONS AND MARKING SCHEME**

**STANDARD OF THE PAPER**

The standard of the Paper is a little higher than the May 2017 diet but lower than the previous Papers ( November 2016, May 2016, November 2015, May 2015) . It is however, similar in format and style to the aforementioned Papers. There were few identifiable ambiguities or typing errors in the paper. The questions were evenly spread over the topics in the syllabus. The marking scheme was straight forward and candidates were rewarded for any meaningful effort.

**PERFORMANCE OF CANDIDATES**

The general performance of candidates can be described as above average. Majority of the candidates who wrote the paper at all centres across the country performed averagely well with many scoring above 40%. Many of these average performers recorded are found in Accra, Kumasi, Takoradi and Cape Coast. The Wa centre recorded four passes, all of an average mark of 50%. The best three candidates who were from the Accra centre scored 82% 80% and 79% respectively. Few candidates scored below 10%. There was no traceable copying by candidates except that some candidates did not number their answers very well, which gave some examiners a tough time trying to separate answered questions for marking and scoring. Few candidates also wasted their limited time trying to solve only three questions for which they ended up scoring high marks on two question and very low marks on one . Most of such candidates did not score the pass mark of 50%. Per the scripts submitted for marking this year's November Examination diet, one will conclude that candidates' preparation for the paper was inadequate and this has reflected in the general performance. In fact, few candidates scored 20/20 on Questions One, Three and Four in this diet's Quantitative Tools in Business Paper.

**NOTABLE STRENGTHS AND WEAKNESSES OF CANDIDATES**

Candidates' notable strengths in the performance were on Algebra (QUESTION ONE), Forecasting (QUESTION THREE) and Statistics (QUESTION FOUR) . Majority of candidates could easily work through simple algebra such as ratios, indices, proportions and its applications i.e. . Simple interest, profit or loss and depreciation. Candidates could also easily form the frequency distribution table, do simple calculation for the mean, median, modal and draw graphs from the frequency table. They could also display points very well on graph papers. These strengths were demonstrated mostly by candidates who took their paper in Accra, Kumasi, Cape Coast and Takoradi. This might be due to the availability of teaching and learning

materials in these centres via the internet, and qualified instructors for teaching the quantitative tools in business. The less patronized questions were QUESTIONS TWO & SEVEN. i.e. Linear Programming & Mathematics of Finance. I suggest ICAG encourage the teaching of the subjects in all the regional capitals.

Candidates' main weakness was inadequate preparation by some candidates for the exams. This is reflected in their inability to do simple computation of indices in QUESTION ONE, and apply the marginal cost and marginal revenue concepts in QUESTION SIX. Many candidates could not interpret figures after they had calculated them. This weakness is common and includes candidates from even the four average high performing centres.

## QUESTION ONE

a) Accra Junction Mall sells apples either for GH¢10 per kg or in bulk GH¢180 per 20 kg. What is the percentage saving per kg from buying in bulk? **(2 marks)**

b) Solve the following expression;

i)  $(X^{-3})^{-4}$

ii)  $\frac{(X^2)^3}{X^5}$

**(3 marks)**

c) A compound house at Ashaiman has a simple meter. There are 5 units' chamber and hall within the compound. The total electricity bill submitted by ECG for the month of September 2017 was GH¢2,700. The outstanding bill on the month of August 2017 was GH¢1,000.

**Required:**

Calculate the bill for each tenant for the month of September 2017 if the tenants had agreed to share the bill in the ratio 2:3:4:5:6. **(5 marks)**

d) The Asogli Company Ltd has been offered the following alternative terms for a one year loan to be repayable plus the interest in full at the end of the year:

i) 2.5% per month compounded monthly

ii) 10% per six months compounded six-monthly.

**Required:**

Calculate, for each alternative the annual percentage rate of interest. Recommend one of the alternatives and explain why. **(10 marks)**

**(Total: 20 marks)**

## QUESTION TWO

An advertising agency wishes to reach two types of audiences: Customers with annual income greater than GH¢ 15,000 (target audience A) and customers with annual income less than GH¢15,000 (target audience B). The total advertising budget is GH¢ 200,000. One programme on TV advertising costs GH¢50,000; one programme on radio advertising costs GH¢20,000. For contract reasons, at least three programmes ought to be on TV, and the number of radio programmes must be limited to five. Surveys indicate that a single TV programme reaches 450,000 customers in target audience A and 50,000 in target audience B. One radio programme reaches 20,000 in target audience A and 80,000 in target audience B.

**Required:**

i) Formulate the linear programming problem.

**(4 marks)**

- ii) Construct the initial simplest tableau. (4 marks)
- iii) Perform the first iteration. (4 marks)
- iv) Determine the media mix to maximize the total reach. (4 marks)
- v) Determine the shadow prices of the binding constraint. (4 marks)

**(Total: 20 marks)**

### QUESTION THREE

A small business is interested in the relationship between the number of hits on its web site (measured by number of visitors that have used the main menu) and the level of web site promotion (in GH¢ 00s). The table below gives the figures for the last six months;

Month	Web Site hits	Web site promotion(GH¢ 00s)
1	25	1.0
2	24	1.2
3	56	1.6
4	54	1.4
5	55	1.2
6	58	1.8

**Required:**

- a) Graph the number of web site hits against web site promotion. (2 marks)
- b) Comment on any possible relationship in (a) above. (2 marks)
- c) Calculate the correlation coefficient and give an interpretation to its value. (5 marks)
- d) Determine the regression line. (5 marks)
- e) Using the regression line found in part (c) above, predict the number of web site hit if the level of monthly promotion were increased to GH¢200. (2 marks)
- f) Comment on the reliability of your prediction in (e) above. (1 mark)
- g) Comment on the simple forecasting model you have developed above. (3 marks)

**(Total: 20 marks)**

#### QUESTION FOUR

Data on access time, in minutes, to a medical consultant who charges by the hour was recorded by his accountant's assistant over a period of one week. Using a class width of 4 minutes, the following frequency distribution table was obtained:

Access Time	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64
Number of Patients	2	5	8	10	13	17	20	16	12	15

**Required:**

- Using a graph sheet draw a less than or a greater than (more than) cumulative frequency curve (ogive) of the data. **(6 marks)**
- From the ogive in (a) above estimate the median access time. **(2 marks)**
- Calculate the mean and the mode of the access times. **(6 marks)**
- Calculate the standard deviation of the distribution. **(4 marks)**
- Using, the values in (b) and (c) above, comment on the skewness of the distribution. **(2 marks)**

**(Total 20 marks)**

#### QUESTION FIVE

The distribution of the total loan amounts defaulted by customers of a bank annually is approximated by a normal distribution. An average default amount is GH¢1.50 million and the standard deviation is GH¢0.50 million.

**Required:**

- Determine the probability that the total loan amount defaulted exceeds GH¢1.50 million. **(3 marks)**
- Determine the probability that the total loan amount defaulted is between GH¢0.86 million and GH¢0.90 million. **(4 marks)**
- Determine the probability that the total amount defaulted is at most GH¢2million. **(4 marks)**
- Determine the amount to be allowed per annum for loan defaults if 1% of actual defaults exceed this amount. **(3 marks)**
- Determine the lower quartile of the distribution of total loan amounts defaulted. **(3 marks)**

f) Determine the upper quartile of the distribution of total loan amounts defaulted. (3 marks)

(Total: 20 marks)

### QUESTION SIX

a) Distinguish between *marginal cost* and *average cost* in production. (2 marks)

b) Kyerewaa Ventures is a manufacturing company in the business of producing beverage Cans for clients in the brewery industry. The weekly total cost to produce  $x$  Cans is given by  $C(x) = 75000 + 100x - 0.03x^2 + 0.000004x^3$ , and the demand function for the Cans is given by  $P(x) = 200 - 0.005x$ .

The company has set production limit to 10,000 Cans and it sells all the Cans that are produced.

#### Required:

i) Derive an expression for marginal cost, marginal revenue and marginal profit. (4 marks)

ii) Determine the cost, revenue and profit when the 2501<sup>st</sup> Can is produced and sold. (3 marks)

iii) Determine the cost, revenue and profit when the 7501<sup>st</sup> Can is produced and sold. (3 marks)

iv) Advise the company whether to produce the 2501<sup>st</sup> Can or the 7501<sup>st</sup> Can. (2 marks)

c) If, after expanding its facilities, the company is capable of producing 60,000 Cans in a day and the total daily cost is given by

$$C(x) = 250,000 + 0.08x + \frac{200,000,000}{x}$$

#### Required:

How many Cans per day should they produce in order to minimize production costs?

(6 marks)

(Totals: 20 marks)

## QUESTION SEVEN

- a) Every Monday, Kwei puts GH¢30 into a savings account at the Ring Bank, which accrues interest of 6.92% per annum compounded weekly.

**Required:**

- i) Draw a cash flow timeline showing the payments, the interest rate and the present values for the first 4 payments. **(3 marks)**
- ii) Determine how long it will take Kwei's account to reach a balance of GH¢4,397.53. Convert your answer into number of years and days to the nearest integer. **(6 marks)**
- iii) Determine how much interest Kwei will receive from the bank during the period of his investment. **(2 marks)**
- b) Alpha Transport Company buys a vehicle for GH¢265,000. The value of the vehicle depreciates on a reducing balance basis at 17% per annum. The company plans to replace this vehicle in 5 years' time and they expect the price of a new vehicle to increase annually by 12%.

**Required:**

- i) Calculate the book value of the vehicle in five years' time. **(3 marks)**
- ii) Determine the amount of money needed in the sinking fund for the company to be able to afford a new vehicle in five years' time. **(3 marks)**
- iii) Calculate the required monthly deposits if the sinking fund earns an interest rate of 11% per annum compounded monthly. **(3 marks)**

**(Total: 20 marks)**

## SOLUTION TO QUESTIONS

### QUESTION ONE

a) 20 kg costs GH¢180. Therefore 1 kg =  $180/20 = \text{GH¢}9$

$$\text{Percentage savings} = \frac{10-9}{20} * 100 \% = 10\%$$

(2 marks)

OR

20 kg @ GH¢10 per kg = GH¢200

Bulk purchase @20 kg=GH¢180

$$\text{Therefore Percentage Savings} = \frac{200-180}{200} * 100 = 10\%$$

b) .

i)  $(X^{-3})^{-4} = X^{(-3*-4)} = X^{12}$

(1.5 marks)

ii)  $\frac{(X^2)^3}{X^5} = X^{(2*3)-5} = X^1 = X$

(1.5 marks)

c)

$2/20 * 1,700$	170
$3/20 * 1,700$	255
$4/20 * 1,700$	340
$5/20 * 1,700$	425
$6/20 * 1,700$	<u>510</u>
	<b>1,700</b>

(5 marks)

d) .

i)  $\text{APR} = (1+0.025)^{12/1} - 1 = 0.34 = 34\%$

(4 marks)

ii)  $\text{APR} = (1+0.10)^{12/6} - 1 = 0.21 = 21\%$

(4 marks)

Because the company is borrowing money, it would prefer a lower interest rate, which is option (ii).

(2 marks)

(Total: 20 marks)



### EXAMINER'S COMMENTS

It was the most popular question among candidates and some candidates who answered it scored extremely high marks (i.e. 20/20,18/20, 17/20). However, Only few candidates understood sub-question (c) and therefore took into account the outstanding balance of GHC 1,000.00 in the calculation. Examiners had to condone on the solution to allow many candidates to secure the pass mark of 10/20 for the question. Please refer to the marking scheme for the best approach to answer it.

### QUESTION TWO

i) Objective Function

$$\text{Maximize } Z = (45,000+5,000)x + (2,000 + 8,000)y$$

$$= 50000x + 10000y$$

$$= 5x + 1y \quad (\text{in GH¢ 0000})$$

Subject to the Constraints

$$\text{Advertising budget } 50,000x + 20,000y \leq 200,000 \quad \text{or} \quad 5x + 2y \leq 20$$

$$\text{Advertising on TV} \quad \quad \quad x \geq 3$$

$$\text{Advertising on Radio} \quad \quad \quad y \leq 5$$

$$\text{Non-negativity} \quad \quad \quad y \geq 0$$

**(4 marks)**

ii) Let  $s_1, s_2, s_3$  be slack variables in the inequalities of the constraints, the LP problem form becomes

$$\text{Maximize } 5x + y - Z = 0$$

Subject to Constraints

$$5x + 2y + s_1 = 20$$

$$x - s_2 = 3$$

$$y + s_3 = 5$$

Solution Variable	Decision Variable		Slack Variable			Solution Quantity
	$x$	$y$	$s_1$	$s_2$	$s_3$	
$s_1$	5	2	1	0	0	20
$s_2$	1	0	0	-1	0	3
$s_3$	0	1	0	0	1	5
$z$	5	1	0	0	0	0

(4 marks)

iii) First iteration

Pivot determination:  $20/5=4$

$$3/1=3$$

Therefore the pivot column is  $x$  under decision variable column and pivot row  $s_2$  under the solution variable. Thus the pivot element is 1.

Solution Variable	Decision Variable		Slack Variable			Solution Quantity
	$x$	$y$	$s_1$	$s_2$	$s_3$	
$s_1$	0	2	1	5	0	5
$x$	1	0	0	-1	0	3
$s_3$	0	1	0	0	1	5
$z$	0	1	0	5	0	-15

(4 marks)

iv) Second iteration

Pivot determination:  $5/5=1$

$$3/-1 = -3 \text{ ignore}$$

$$5/0 = \infty \text{ ignore.}$$

Therefore the pivot element is 5.

Solution Variable	Decision Variable		Slack Variable			Solution Quantity
	$x$	$y$	$s_1$	$s_2$	$s_3$	
$s_2$	0	$2/5$	$1/5$	1	0	1
$x$	1	$2/5$	$1/5$	0	0	4

$s_3$	0	1	0	0	1	5
$z$	0	-2	-1	0	0	-20

As the contribution row has no positive values we have reached our optimal solution and

$$x = 4, y = 0.$$

Total audience reach = 20,000. **(4 marks)**

v) Shadow prices the three binding constraints are 1, 0 and 0 respectively. **(4 marks)**

**(Total: 20 marks)**

#### EXAMINER'S COMMENTS

This question was a less popular question among candidates, and those who answered it did not do well with the average mark hovering around the figure 3/20. Candidates who attempted this question did not understand the question and those who could formulate the linear programming problems could not solve by the simplex method. Please refer to the marking scheme for the best approach to answer it.

**QUESTION THREE**

- a)  
 b) The relationship may be described as linear. **(2 marks)**  
 c)

Month	y	x	xy	y <sup>2</sup>	x <sup>2</sup>
1	25	1.0	25.0	625	1.00
2	24	1.2	28.8	576	1.44
3	56	1.6	89.6	3136	2.56
4	54	1.4	75.6	2916	1.96
5	55	1.2	66.0	3025	1.44
6	58	1.8	104.4	3364	3.24
$\Sigma$	272	8.2	389.4	13,642	11.64

$$\begin{aligned}
 r &= \frac{6 \times 389.4 - 272 \times 8.2}{\sqrt{[6 \times 13642 - (272)^2][6 \times 11.64 - (8.2)^2]}} = \frac{2336.4 - 2230.4}{\sqrt{[81852 - 73984][69.84 - 67.24]}} \\
 &= \frac{106}{\sqrt{7868 \times 2.6}} \\
 &= \frac{106}{\sqrt{20456.8}} \\
 &= 0.7411174
 \end{aligned}$$

There exist strong positive correlation website hits and website promotions.

**(5 marks)**

ALT

y	x	R <sub>y</sub>	R <sub>x</sub>	d = R <sub>x</sub> - R <sub>y</sub>	d <sup>2</sup>
25	1.0	5	6	1	1
24	1.2	6	4.5	-1.5	2.25
56	1.6	2	2	0	0
54	1.4	4	3	-1	1
55	1.2	3	4.5	1.5	2.25
58	1.8	1	1	0	0

For tie in rank,

$$r_k = 1 - \frac{6 \left[ \sum d^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2) + \dots \right]}{N^3 - N}$$

d is the difference in rank

N is the number of observations

m<sub>i</sub> is number of items of equal ranks i

$$r = 1 - \frac{6 \left[ 6.5 + \frac{1}{12}(2^3 - 2) \right]}{6^3 - 6}$$

$$r = 1 - \frac{6(7)}{210}$$

$$r = 1 - \frac{42}{210}$$

$$r = 1 - 0.2$$

$$r = 0.8$$

d)

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$b = \frac{6 \times 38940 - 820 \times 272}{6 \times 116400 - (820)^2} = \frac{233640 - 223040}{698400 - 672400} = \frac{53}{130} = 0.40769$$

$$a = \frac{272}{6} - (0.40769) \times \frac{820}{6} = -10.3843 = -10.38$$

Therefore the regression equation is  $y = -10.38 + 0.40769x$ .

**(5 marks)**

e) When  $x = 200$

$$y = -10.38 + 0.40769(200)$$

$$y = 71.158$$

$$y = 71$$

**(2 marks)**

Therefore the number of hits will be 71 given that the level of monthly promotion is increased to 200.

f) The value for  $x$  used for prediction is relatively outside the range. This is an extrapolation and the prediction may not be entirely accurate. Also, the number of data point is few for accurate predictions.

**(1 mark)**

g) The coefficient of determination  $D = r^2 = 0.74^2 = 0.5476$ . This implies 54.76 percent of variations in web site promotions is explained by web site hits and the remaining 45.24 percent is explained by other factors.

**(3 marks)**

**(Total: 20 marks)**

### EXAMINER'S COMMENTS

Interestingly, Question Three was the second most popular question among candidates after Questions Four and One, with also some candidates scoring 20/20. However, few candidates could manage 10 marks and above. Many candidates who attempted this question could not distinguish between the dependent variable; web site promotion and the independent variable; web site hits. This led to candidates not getting many marks for accuracy in the suggested marking scheme. I would describe candidates performance on the question as below average.

## QUESTION FOUR

a)

Class Interval	Frequency f	Class Boundary	Cumulative Frequency	Reverse CF	Midpoint x	fx	fx <sup>2</sup>
15-19	2	14.5-19.5	2	118	17	34	578
20-24	5	19.5-24.5	7	116	22	110	2,420
25-29	8	24.5-29.5	15	111	27	216	5,832
30-34	10	29.5-34.5	25	103	32	320	10,240
35-39	13	34.5-39.5	38	93	37	481	17,797
40-44	17	39.5-44.5	55	80	42	714	29,988
45-49	20	44.5-49.5	75	63	47	940	44,180
50-54	16	49.5-54.5	91	43	52	832	43,264
55-59	12	54.5-59.5	103	27	57	684	38,988
60-64	15	59.5-64.5	118	15	62	930	57,660
	<b>118</b>					<b>5,261</b>	<b>250,947</b>

(6 marks)

b) From the graph of the ogives the median access time is 45.5minutes

(2 marks)

$$c) \text{ Mean} = \frac{\sum fx}{\sum f} = \frac{5,261}{118} = 44.58 \text{ minutes}$$

$$\text{Mode} = LB + \left[ \frac{D_1}{D_1 + D_2} \right] C_m \quad C_m = 44.5 - 49.5 = 5; \quad D_1 = 20 - 17 = 3; \quad D_2 = 20 - 16 = 4$$

$$\text{Mode} = 44.5 + \left[ \frac{3}{3+4} \right] \times 5 = 46.64 \text{ minutes}$$

(6 marks)

d)

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left[ \frac{\sum fx}{\sum f} \right]^2}$$

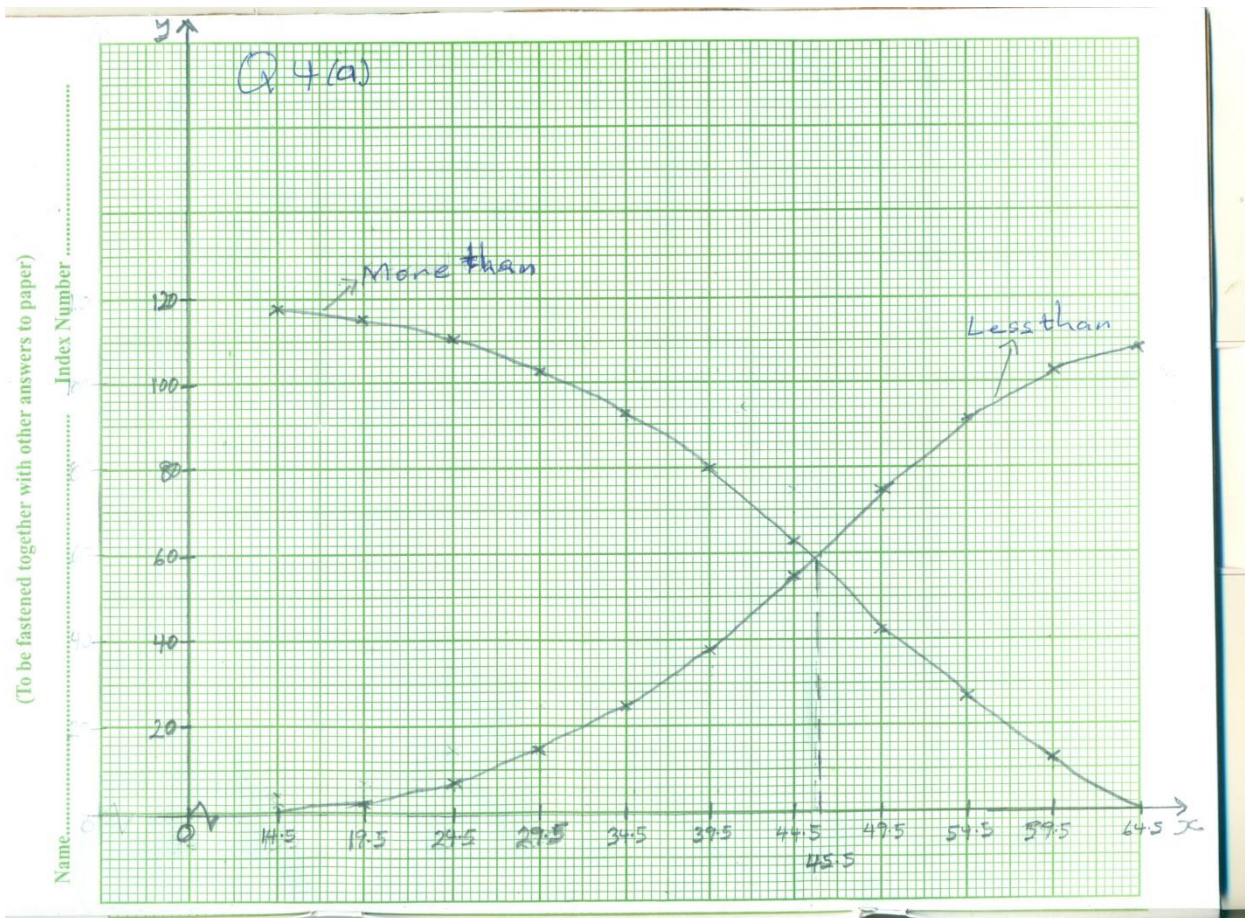
$$= \sqrt{\frac{250,947 - 44.58^2}{118}} = \sqrt{2126.67 - 1987.38}$$

$$= \sqrt{139.29} = 11.8 \text{ minutes}$$

(4 marks)

e)

Since the Mode = 46.64 > Median = 45.5 > Mean = 44.58, the distribution is left skewed or negatively skewed.



(2 marks)

(Total: 20 marks)



### EXAMINER'S COMMENTS

It was the third most popular question among candidates after Question One and most candidates who attempted it performed extremely well with many scoring (20/20, 19/20, 18/20, 18/20). Candidates could easily draw a less than or a greater ogive and compute summary measures for the data. In fact candidates could easily give valid comments on the distribution of the data using the summary measures.

### QUESTION FIVE

Let  $X$  be a random variable depicting total loan amounts defaulted by customers of the bank annually. Then  $X \sim (1.5, 0.25)$

- a) Since any normal distribution is symmetric about the mean, the probability that  $X$  takes on values greater than its mean is 0.5.

$$\text{I.e. } P(X \geq 1.5) = 0.5$$

**(3 marks)**

b)

$$P(0.86 \leq X \leq 0.9)$$

$$Z = \frac{X - \mu}{\delta}$$

$$Z_1 = \frac{0.86 - 1.5}{0.5} = -1.28$$

$$Z_2 = \frac{0.9 - 1.5}{0.5} = -1.2$$

$$P(-1.28 \leq Z \leq -1.2) = P(0 \leq Z \leq 1.28) - P(0 \leq Z \leq 1.2)$$

$$0.3997 - 0.3849 = 0.0148$$

**(4 marks)**

c)

$$P(X \leq 2)$$

$$Z_1 = \frac{2 - 1.5}{0.5} = 1$$

$$\begin{aligned} P(Z \leq 1) &= 0.5 + P(0 \leq Z \leq 1) \\ &= 0.5 + 0.3413 \end{aligned}$$

$$= 0.8413$$

(4 marks)

d)

$$P(Z \geq Z_1) = 0.01$$

$$P(0 \leq Z \leq Z_1) = 0.49$$

$$Z_1 = 2.33$$

$$X = z\delta + \mu$$

$$X = 2.33(0.5) + 1.5 = 2.665$$

Ie GH¢2.665million

(3 marks)

e)

The lower quartile is the same as the 25<sup>th</sup> percentile of the distribution:

$$P(0 \leq Z \leq Z_1) = 0.25$$

$$Z_1 = -0.67$$

$$X = -0.67(0.5) + 1.5$$

$$= 1.165$$

The lower quartile is GH¢1.165million

(3 marks)

f)

Upper quartile is the same as the 75<sup>th</sup> percentile of the distribution:

$$0.5 + P(0 \leq Z \leq Z_1)$$

$$Z_1 = 0.67$$

$$X = 0.67(0.5) + 1.5$$

$$= 1.835$$

The upper quartile is GH¢1.835million.

(3 marks)

(Total: 20 marks)

### EXAMINER'S COMMENTS

This question was the next most popular after question four in this diet. Candidates were well-prepared for normal distribution question. However, many candidates could not answer sub-question (e), (f) and (g) which involves inverse reading of values from the normal distribution table. In particular, looking for the lower quartile and upper quartile values of the distribution was problematic for students. Note, some candidates scored 0 on this question.

I encourage candidates who will want to answer normal distribution questions to master the inverse reading of values from the table as well.

## QUESTION SIX

a)

Marginal cost is the change in total cost when another unit is produced.

Average cost is the total cost divided by the number of goods produced.

Alternatively:

Marginal cost refers to the additional amount required to produce the  $x+1$ th unit.

The average cost is the total cost of production of  $x$  units of the product divided by  $x$ .

**(2 marks)**

b)

i)

If Total Cost,  $C(x) = 75000 + 100x - 0.03x^2 + 0.000004x^3$  and  $P(x) = 200 - 0.005x$

Marginal Cost,  $MC = \frac{dC}{dx} = 100 - 0.06x + 0.000012x^2$

Total Revenue,  $TR = xP(x) = x(200 - 0.005x) = 200x - 0.005x^2$

Marginal Revenue,  $MR = \frac{dR(x)}{dx} = 200 - 0.01x$

Profit,  $\pi = TR - TC = 200x - 0.005x^2 - (75000 + 100x - 0.03x^2 + 0.000004x^3)$

$$\pi = -0.000004x^3 + 0.025x^2 + 100x - 75000$$

Marginal Profit,  $\frac{d\pi}{dx} = -0.000012x^2 + 0.05x + 100$

**(4 marks)**

ii) The marginal functions when 2500 cans are sold are:

$$\begin{aligned} MC(2500) &= 100 - 0.06(2500) + 0.000012(2500)^2 \\ &= 25 \end{aligned}$$

$$\begin{aligned} MR(2500) &= 200 - 0.01(2500) \\ &= 175 \end{aligned}$$

$$\begin{aligned} MP(2500) &= -0.000012(2500)^2 + 0.05(2500) + 100 \\ &= 150 \end{aligned}$$

**(3 marks)**

iii)

The marginal functions when 7500 cans are sold are:

$$\begin{aligned} MC(7500) &= 100 - 0.06(7500) + 0.000012(7500)^2 \\ &= 125 \end{aligned}$$

$$\begin{aligned} MR(7500) &= 200 - 0.01(7500) \\ &= 175 \end{aligned}$$

$$MP(7500) = -0.000012(7500)^2 + 0.05(7500) + 100$$

$$= -200$$

(3 marks)

iv)

Upon producing and selling the 2501st can it will cost the company approximately GH¢25 to produce the can and they will see an added GH¢175 in revenue and GH¢150 in profit. However, when they produce and sell the 7501st can it will cost an additional GH¢325 and they will receive an extra GH¢125 in revenue, but make a loss of GH¢200.

The company should produce the 2501<sup>st</sup> can.

(2 marks)

c)

$$C(x) = 250,000 + 0.08x + \frac{200,000,000}{x}$$

$$\frac{dC}{dx} = 0.08 - \frac{200,000,000}{x^2}$$

For maximum or minimum value of the function;  $\frac{dC}{dx} = 0$

$$0.08 - \frac{200,000,000}{x^2} = 0$$

$$0.08x^2 = 200,000,000$$

$$x^2 = 2,500,000,000$$

$$x = \pm 50,000$$

Since x cannot be negative the relevant solution is  $x = 50,000$

Using the second derivative to test the nature of the turning point;

$$\frac{d^2C}{dx^2} = \frac{400,000,000}{x^3}$$

The second derivative is positive for all values of  $x > 0$  and so, in the range of possible solutions the function is always concave up and so producing 50,000 cans per day will minimize production cost.

(2 marks)

(Total: 20 marks)

### EXAMINER'S COMMENTS

Question Six was one of the least popular questions among candidates but most candidates who attempted this question did averagely well. Many candidates who attempted this question scored low marks with a few getting above average of 10. There were few candidates who recorded 0/20. The effect of candidates inability to comprehend the concept of marginal cost and marginal revenue was clearly demonstrated in the answers provided by candidates. Only few candidates could manage with the computation of profit from the cost and revenue functions.

## QUESTION SEVEN

i)

Time (week)	$i=0.133\%$	1	2	3	4
		↓	↓	↓	↓
Payment (GH¢)		-30	-30	-30	-30
P V (GH¢)		30	29.96	29.92	29.88

Interest rate per week,  $i = \frac{0.0692}{52} = 0.00133$

Week1: PV = GH¢30

Week 2: PV =  $\frac{30}{1.00133} = \text{GH¢}29.96$

Week 3: PV =  $\frac{30}{1.00133^2} = \text{GH¢}29.92$

Week 4: PV =  $\frac{30}{1.00133^3} = \text{GH¢}29.88$

**(3 marks)**

ii)

$$FV = \frac{A[(1+i)^n - 1]}{i}$$

$A=30$ ,  $FV=4397.53$ ,  $i=6.92\%$  per annum

$$4397.53 = \frac{30 \left[ 1 + \frac{0.0692}{52} \right]^{52 \times n} - 1}{\frac{0.0692}{52}}$$

$$4397.53 = \frac{30[(1.00133)^{52n} - 1]}{0.00133}$$

$$\frac{4397.53 \times 0.00133}{30} = (1.00133)^{52n} - 1$$

$$0.195 + 1 = (1.00133)^{52n}$$

Taking log of both sides:

$$52n \log 1.00133 = \log 1.195$$

$$n = \frac{0.07737}{0.0005772 \times 52} = 2.578 \text{ years}$$

Convert 0.578 years into days:

$$\text{ie } 0.578 \times 365 = 210.97 \text{ days.}$$

Thus the required period is 2 years and 211 days.

**(6 marks)**

iii)

The total amount Kwei invests is:

$$30 \times 52 \times 2.578 = 4021.68$$

Therefore, the total amount of interest received by Kwei:

$$4397.53 - 4021.68 = 375.85$$

ie GH¢375.85

**(2 marks)**

b)

i)

$$A = P(1+i)^n$$

$$P = 265000; \quad i = 0.17; \quad n = 5; \quad A = ?$$

$$A = 265000(1-0.17)^5$$

$$= 104384.58$$

ie GH¢104384.58

**(3 marks)**

ii)

$$P = 265000; \quad i = 0.12; \quad n = 5; \quad A = ?$$

$$A = P(1+i)^n$$

$$= 265000(1+0.12)^5$$

$$467020.55$$

Therefore, the balance of the sinking fund must be greater than the cost of a new vehicle in 5 years' time less the money from the sale of the old vehicle:

$$467020.55 - 104384.58 \\ = \text{GH}\text{c}362635.97$$

**(3 marks)**

iii)

Required monthly payment into the sinking fund:

$$FV = 362635.97; i = 0.11/12 = 0.00917; n = 5 \times 12 = 60$$

$$\text{From } FV = \frac{A[(1+i)^n - 1]}{i}$$

$$A = \frac{FV \times i}{[(1+i)^n - 1]}$$

$$A = \frac{362635.97 \times 0.00917}{[(1+0.00917)^{60} - 1]}$$

$$A = 4559.92$$

Thus the company must deposit GHc4,560 each month.

**(3 marks)**

**(Total: 20 marks)**

### EXAMINER'S COMMENTS

Answers provided by candidates for Question Seven were not appropriate. Candidates could not draw simple time line. The concepts of sinking fund, present value, interest rate was really a problem for candidates. I recommend the topic is treated well by course lecturers as it is more of applied mathematics.