MAY 2016 PROFESSIONAL EXAMINATION
QUANTITATIVE TOOLS IN BUSINESS (1.4)
EXAMINER'S REPORT, QUESTIONS AND MARKING SCHEME

EXAMINER'S REPORT

STANDARD OF THE PAPER

The Quantitative Tools in Business, Paper 1.4, taken in May was generally well written and the questions adequately cover the Level One Syllabus and the ICAG Manual.

The standard of the Paper is comparable to the November 2015 Paper but it is a little different in format and style from the previously administered Papers. The marking scheme was well-drawn; every sub-question had marks duly allocated and the marks were adequate for each question (i.e. it followed the weighting in the revised syllabus). Finally, I will like to say that the questions were evenly spread over the topics in the syllabus. The only limitations of the paper are inadequate hints on Question Two (ii) and the preamble of Question Three. However, the marking scheme was straight forward and candidates were rewarded for any meaningful comment or explanation.

GENERAL PERFORMANCE OF CANDIDATES

The general performance of candidates can be described as below average. Majority of the candidates who wrote the paper at centres outside Accra, Kumasi and Cape Coast performed extremely poorly with a few scoring above 40%. The few high performers recorded are found in Accra, Kumasi and Cape Coast. Indeed the best candidate scored 89% and he wrote the paper at the Cape Coast centre. About half-dozen of candidates scored between 0% and 2%. There were no traceable copying by candidates except that some candidates decided to copy the questions into their booklets before solving them. Majority of candidates also wasted their limited time trying to explain or give interpretation of symbols in the formulae before use. There were a lot of bad numbering of questions and examiners went through some difficulties trying to separate answered questions for marking and scoring. Per the scripts submitted for marking this year's May Examination diet, one will conclude that candidates were not adequately prepared for the paper and this has reflected in the general performance. In fact, few candidates decided to present blank booklets for marking in this diet.

QUANTITATIVE TOOLS IN BUSINESS QUESTIONS

QUESTION ONE

Mr. Boo, a Chartered Accountant is investigating the distribution of number of fuel gallons consumed per day in the year 2010. He conducted a survey on the number of fuel gallons consumed per day in the year 2010 and recorded the following results:

NUMBER OF FUEL GALLONS	PERCENTAGE OF CONSUMPTION
0 but under 5	38
5 but under 10	25
10 but under 15	16
15 but under 20	21

Required:
a) Calculate the following measures of central tendency:
i) Mean **(3 marks)**
ii) Median **(3 marks)**
iii) Mode **(3 marks)**

b) Compute the following measures of dispersion and skewness
i) Inter-Quartile **(3 marks)**
ii) Variance **(3 marks)**
iii) Appropriate coefficient of skewness. **(3 marks)**

c) Using (a) and (b) above, comment on the distribution of the number of fuel gallons consumed per day in 2010. **(2 marks)**

(Total: 20 marks)

QUESTION TWO

a) Countess Company trades in bars of Golden Tree Chocolate from Tema Cocoa processing company limited. The number of bars of chocolate sold per quarter over a four-year period by Countess Company is;

YEAR	QUARTER 1	QUARTER 2	QUARTER 3	QUARTER 4
1	20	10	4	11
2	33	17	9	18
3	45	23	11	25
4	60	30	13	29

Required:
i) Plot the data on a graph. **(3 marks)**
ii) Calculate a linear regression trend equation for the data. **(5 marks)**
iii) Calculate the four seasonal components using a multiplicative model. **(5 marks)**

iv) Forecast the number of bars of chocolate for the next two years. **(5 marks)**
v) Comment on the reliability of years forecasts in iv) above. **(2 marks)**
(Total: 20marks)

QUESTION THREE

a) If from a normal pack of 52 cards, consisting of four suites each of 13 cards, one card is randomly selected;

Required:
Calculate the probabilities of selecting the following:

i) An ace (**2 marks**)
ii) A club (**2 marks**)
iii) An ace or a club (**2 marks**)
iv) The ace of clubs (**2 marks**)
v) A picture card(i.e. a jack, queen or king) (**2 marks**)
vi) A red card (**2 marks**)
vii) A red king (**2 marks**)
viii) A red picture card (**2marks**

b) Given that a card selected is red, calculate the probability that is a picture card. **(4 marks)**
(Total: 20 marks)

QUESTION FOUR

a) Receipts at a particular depot have amounts which follow the Normal distribution with mean of GH¢103.60 and a standard deviation of GH¢8.75.

Required:
i) Determine the percentage of receipts over GH¢ 120.05. **(3 marks)**
ii) Determine the percentage of receipts below GH¢ 92.75. **(3 marks)**
iii) Determine the percentage of receipts between GH¢ 83.65 and GHS 117.60. **(3 marks)**
iv) Determine the receipts amount such that approximately 25 percent of receipts are greater amounts. **(3 marks)**
v) Above what amount will 90 percent of receipts lie? **(3 marks)**

b) If 10.56 percent of receipts have an amount above 110.05 and 4.01 percent of receipts have an amount above 120.05.
Required:
Calculate, the mean and standard deviation of the receipts assuming that they are normally distributed. **(5 marks)**
(Total: 20 marks)

QUESTIONS FIVE

a) If the total cost (in Ghana cedis) of producing x items is C(x) =2600 +2x+0.001x²;

Required:
i) Calculate, the total cost, average cost, and marginal cost of producing 5000 items. **(6 marks)**
ii) Determine the production level at which the average cost will be the lowest. **(4 marks)**

b) Mini has some GH¢10 notes and some GH¢20 notes. If she has 273 notes worth a total of GH¢4,370;

Required:
i)   Write down the system of linear equations. **(5 marks)**
ii)   Solve the system of linear equations. **(5 marks)**

(Total: 20 marks)

QUESTION SIX

a) i) Distinguish between a Sample and a Population. **(2 marks)**

   ii) Distinguish between a Statistic and a Parameter. **(2 marks)**

b) Use the sample data below to answer the following questions:

$$\sum_{i=1}^{n} X_i = 220 \qquad \sum_{i=1}^{n} (X_i - \overline{X})^2 = 440 \qquad \sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -568$$

$$\sum_{i=1}^{n} Y_i = 385 \qquad \sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 1120 \qquad n = 11$$

Required:
i)   Calculate the sample mean, variance, and standard deviation for X. **(5 marks)**
ii)   Calculate the sample mean, variance and standard deviation for Y. **(5 marks)**
iii)   Calculate the sample covariance between X and Y. **(3 marks)**
iv)   Calculate the sample correlation between X and Y. **(3 marks)**

(Total: 20 marks)

QUESTION SEVEN

A loan of GH¢3,000 at an effective annual interest rate of i = 2% is amortized by means of 12 annual payments, beginning a year after the loan is taken.

Hint: The schedule should have the following columns: Payment, Interest Due, Principal Repaid and Outstanding Balance.

Required:
Construct an amortization schedule. **(Total: 20 marks)**

SCHEME OF QUANTITATIVE TOOLS IN BUSINESS

QUESTION ONE

(a) & (b)

Classes	Class Boundaries	Class Midpoints x	Frequency f	fx	fx^2
0-4	-0.5-4.5	2	38	76	152
5-9	4.5-9.5	7	25	175	1,225
10-14	9.5-14.5	12	16	192	2,304
15-19	14.5-19.5	17	21	357	6,069
$\sum$			100	800	9,796

(a)

(i) $\bar{X} = \dfrac{800}{100} = 8$ Gallons

(ii) $Median = 4.5 + \left(\dfrac{\dfrac{100}{2} - 38}{25} \right) \times 5 = 4.5 + 2.4 = 6.9$ Gallon

Median number of fuel Gallons is 7 Gallons

(iii) $Model = -0.5 + \left(\dfrac{38}{38 + 13} \right) \times 5 = -0.5 + 5 \times 0.745 = 3.225$. Hence the mode number of fuel Gallons is 4.

(b)

(i) $q_1 = -0.5 + \left(\dfrac{25 - 0}{38} \right) \times 5 = 0.16$

$q_3 = 9.5. + \left(\dfrac{75 - 63}{16} \right) \times 5 = 13.25$

$InterQuartileRange = 13.25 - 0.16 = 13.09$

Inter Quartile Range Number of fuel Gallons is 13.09

(ii) $Variance = \dfrac{1}{99} \left\{ 9750 - \dfrac{800^2}{100} \right\} = \dfrac{3350}{99} = 33.84$

Variance Number of fuel Gallons is 33.84

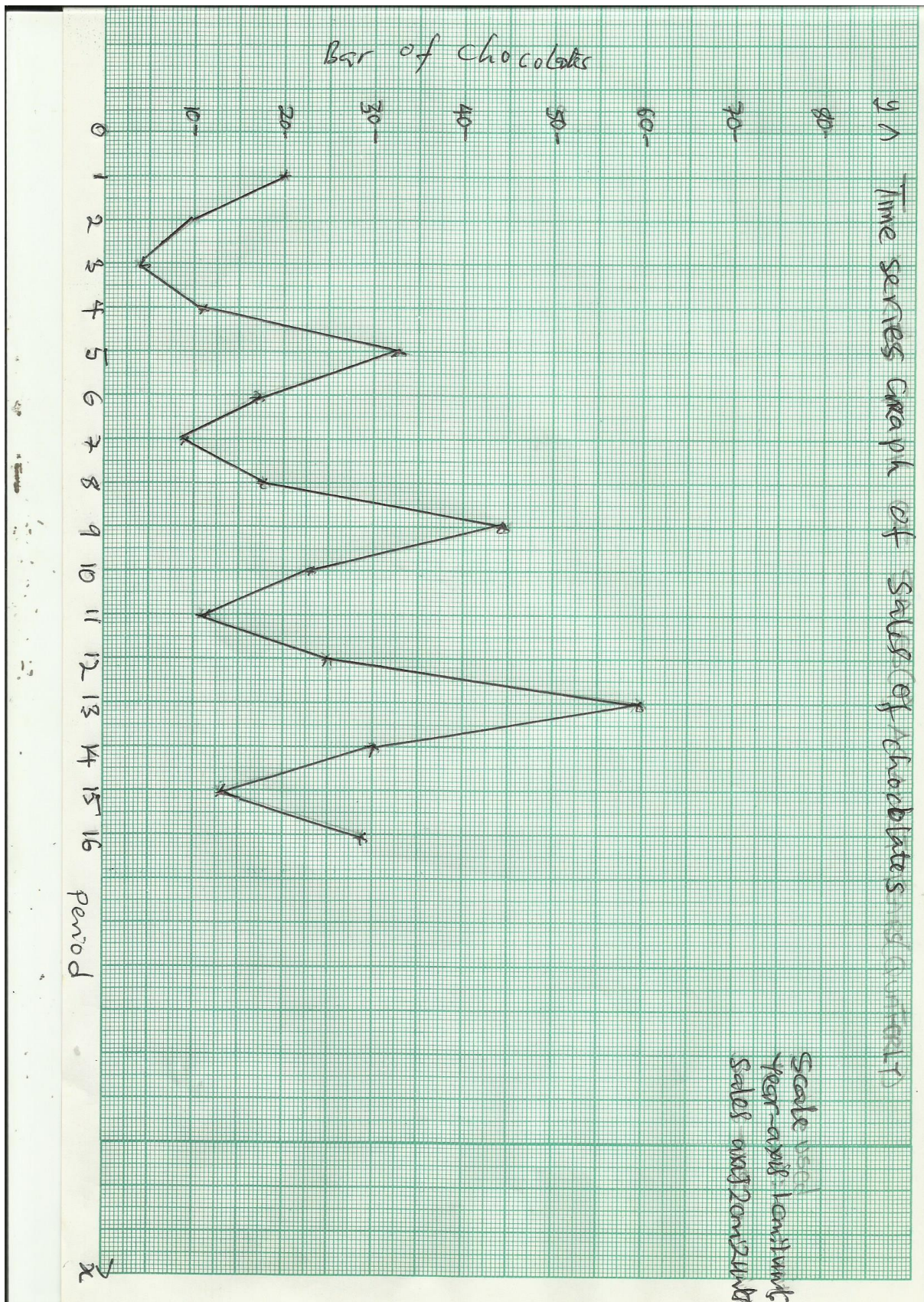(iii) $Skewness = \dfrac{3(8 - 4)}{5.80} = \dfrac{12}{5.80} = 2.07$

(c) It is a unimodal data. Since $4 < 7 < 8$ and the coefficient of skewness is positive the data can be described as positively skewed.

EXAMINER'S COMMENTS

It was the most popular question among candidates but many candidates who answered it scored extremely low marks (i.e. 0/20,1/20, 1.5/20). Only a handful of candidates could come up with the correct classes for number of fuel gallons to correctly answer sub-questions (a), (b) and hence (c) for the various marks allocated to the individual sub-questions. Please refer to the marking scheme for the best approach to answer it.

QUESTION TWO

i)

Time series Graph of Sales of chocolates (QUARTERLY)

Bar of chocolates

Y-axis (Bar of chocolates): 0, 10, 20, 30, 40, 50, 60, 70, 80

X-axis (Period): 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Scale used
Year-axis: 1cm : 1 unit
Sales axis: 1cm : 2 units

ii)

Year x	Sales y	xy	x^2	Trend line $y = 11.33 + 1.3x$	
1	20	20	1	12.625	158.42
2	10	20	4	13.925	71.81
3	4	12	9	15.225	26.27
4	11	44	16	16.525	66.57
5	33	165	25	17.825	185.13
6	17	102	36	19.125	88.89
7	9	63	49	20.425	44.06
8	18	144	64	21.725	82.85
9	45	405	81	23.025	195.44
10	23	230	100	24.325	94.55
11	11	121	121	25.625	42.93
12	25	300	144	26.925	92.85
13	60	780	169	28.225	212.58
14	30	420	196	29.525	101.61
15	13	195	225	30.825	42.17
16	29	464	256	32.125	90.27
136	358	3485	1496		

The trend line is given by $y = a + bx$

$$b = \frac{16 \times 3485 - 136 \times 358}{16 \times 1496 - (136)^2} = \frac{55760 - 48688}{23936 - 18496} = \frac{7072}{5440} = 1.3$$

$$a = \frac{358}{16} - 1.3\left(\frac{136}{16}\right) = 122.375 - 11.05 = 11.325$$

Therefore the trendline is $y = 11.325 + 1.3x$.

iii)

	Q1	Q2	Q3	Q4	
	158.42				
		71.81	26.27	66.57	
	185.13				
		88.89	44.06	82.85	
	195.44				
		94.55	42.93	92.85	
	212.58				
		101.61	42.17	90.27	
Av. Sea. Value	187.89	89.22	38.86	83.14	399.10

Adjusted Av.Sea.Value	188.31	89.42	38.95		83.32	400

$$187.89 \times \frac{400}{399.10} \qquad 89.22 \times \frac{400}{399.10} \qquad 38.86 \times \frac{400}{399.10}$$

$$83.14 \times \frac{400}{399.10}$$

iv)

Year 5	Trendline	Trendline $\times$ Av. Seasonal value	Seasonally adjusted forecast
17	33.43	$33.43 \times 1.878.31$	62.95
18	34.73	34.73×89.42	31.05
19	36.03	36.03×0.3885	14.03
20	37.33	$37.33 \times 0.83.32$	31.10
21	38.63	$38.63 \times 1.878.31$	72.75
22	39.93	39.93×89.42	35.70
23	41.23	41.23×0.3885	16.06
24	42.53	$42.53 \times 0.83.32$	35.44

Forecast values in iv) above are not reliable because they are extrapolations. i.e. forecast beyond the range of the data.

EXAMINER'S COMMENTS
This question was the next popularly answered question by candidates, and those who answered it did very well with the average mark hovering around the figure 11/20 and one candidate scoring 20/20. However, many candidates tried to compute a moving average trend estimate in sub-question (ii) instead of linear regression trend estimate. Some candidates who managed to get the correct forecast in sub-question (iv) could not give a very good comment in sub-question (v).

QUESTION THREE

a) Sample space = 52 cards

    i) $P(\text{an ace}) = \dfrac{4}{52} = \dfrac{1}{13}$

    ii) $Pr(A \text{ club}) = \dfrac{13}{52} = \dfrac{1}{4}$

    iii) $Pr(\text{An ace or a club}) = \dfrac{4}{52} + \dfrac{13}{52} - \dfrac{1}{52} = \dfrac{4+13-1}{52} = \dfrac{16}{52} = \dfrac{4}{13}$

    iv) $Pr(\text{The ace of clubs}) = \dfrac{1}{52}$

    v) $Pr(A \text{ picture card}) = \dfrac{12}{52} = \dfrac{3}{13}$

    vi) $Pr(A \text{ red card}) = \dfrac{26}{52} = \dfrac{1}{2}$

    vii) $Pr(A \text{ red king}) = \dfrac{2}{52} = \dfrac{1}{26}$

    viii) $Pr(A \text{ red picture card}) = \dfrac{6}{52} = \dfrac{3}{26}$

b) $Pr(\text{picture card} \mid \text{red card}) = \dfrac{6}{26} = \dfrac{3}{13}$

EXAMINER'S COMMENTS

Interestedly Question Three was the third most popular question among candidates after Question Two and with also one candidate scored 20/20. However, few candidates could manage 15 marks and above. Some did not know the experiment of cards shuffling and those who knew it could not compute the probabilities, even with the little hint given in the preamble of the question. Almost all high performers of this diet's Paper 1.4 answered this question.
Candidates who intend to answer questions on Probability in any sitting are encouraged to master the three main experiments of Coin flipping, Die Rolling and Card shuffling.

QUESTION FOUR

Let X be normally distributed with mean 103.60 and standard deviation 8.75.

(i)

$$\Pr(X > 120.05) = \Pr\left(\frac{X - 103.6}{8.75} > \frac{120.05 - 103.6}{8.75}\right)$$
$$= \Pr(Z > 1.88)$$
$$= 0.5 - 0.4699$$
$$= 0.0301$$

About 3% of the receipts is over GHC 120.05

(ii)

$$\Pr(X < 92.75) = \Pr\left(\frac{X - 103.6}{8.75} < \frac{92.75 - 103.6}{8.75}\right)$$
$$= \Pr(Z < -1.24)$$
$$= 0.5 - 0.3925$$
$$= 0.1075$$

About 10.75% of the receipts is below GHC 92.75

(iii)

$$\Pr(83.65 < X < 117) = \Pr\left(\frac{83.65 - 103.6}{8.75} < \frac{X - 103.6}{8.75} < \frac{117.60 - 103.6}{8.75}\right)$$
$$= \Pr(-2.28 < Z < 1.60)$$
$$= 0.4452 + 0.4980$$
$$= 0.9432$$

About 94.32% of the receipts lies between GHC 83.65 and GHC 117.00

(iv)

$$\Pr(X > k) = 0.25$$
$$\Pr\left(Z > \frac{k - 103.60}{8.75}\right) = 0.25$$
$$\Pr(Z > c) = 0.25$$
$$c = \frac{k - 103.60}{8.75}$$

$$\Pr(Z \le c) = 0.75$$

$$c = 0.67$$

$$\frac{k - 103.6}{8.75} = 0.67$$

$$k = 0.67(0875) + 103.6 = 109.102$$

(i)

$$\Pr(X > k) = 0.90$$

$$\Pr\left(Z > \frac{k - 103.60}{8.75}\right) = 0.25$$

$$\Pr(Z > c) = 0.90$$

$$c = \frac{k - 103.60}{8.75}$$

$$\Pr(Z \le c) = 0.1$$

$$c = -0.128$$

$$\frac{k - 103.60}{8.75} = -0.128$$

$$k = -0.128(8.75) + 103.60 = 92.40$$

b)

$$\Pr\left(Z > \frac{110.5 - \mu}{\sigma}\right) = 0.1056$$

$$\Pr(Z > k) = 0.1056$$

$$k = \frac{110.5 - \mu}{\sigma}$$

$$\Pr(Z \le k) = 0.8944$$

$$\frac{110.5 - \mu}{\sigma} = 1.25$$

$$\mu + 1.25\sigma = 110.05 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(1)$$

$$\Pr\left(Z > \frac{120.05 - \mu}{\sigma}\right) = 0.041$$

$$\Pr(Z > k) = 0.041$$

$$k = \frac{120.05 - \mu}{\sigma}$$

$$\Pr(Z \le k) = 0.9599$$

$$\frac{110.5 - \mu}{\sigma} = 1.75$$

$$\mu + 1.75\sigma = 120.05 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(2)$$

Solve equation (1) and two simultaneously for the parameters as follows:

Subtract equation (1) from (2) $0.5\sigma = 10 \Rightarrow \sigma = \dfrac{10}{0.5} = 20.00$. Substitute $\sigma = 20$ in the equation (10) we have

$\mu + 1.25(20) = 110.5$
$\mu = 110.5 - 25$
$\mu = 85.05$

EXAMINER'S COMMENTS
Candidates who answered this question performed well. In fact, they could easily read from the normal distribution table probabilities, hence questions (a) (i)-(v) were well-answered. However, some candidates could not do the inverse reading of probabilities from the normal distribution table and hence form correctly the two equations with the two unknown variables the mean and standard deviation. Some candidates who could form the equations could manage a good solution.

QUESTION FIVE

(a) Cost : $C(x) = 2600 + 2x + 0.001x^2$

    (i) $x = 5000$

        Total Cost

        $C(5000) = 2600 + 2(5000) + 0.001(5000)^2 = 2600 + 10000 + 25000 = 37600$

        Average cost=37600/5000=7.52

        Marginal Cost= $\dfrac{dC(x)}{dx} = 2 + 0.002x$

        Marginal Cost=2+0.002(5000)=12

    (ii) $Ac = \dfrac{2600 + 2x + 0.001x}{x}$

        Average Cost $= 0.001 - 2600x^{-2} = 0$

        Therefore we have

        $x^2 = 2600 / 0.001 = 2,600,000 \Rightarrow x = \sqrt{2,600,000} = 1612.452$ items

(b) Let M be number of 10 GHS notes and N be number of 20 GHS notes

    (i)

        $M + N = 273$
        $10M + 20N = 4370$

    (ii)

$$M = \frac{\begin{vmatrix} 273 & 1 \\ 4370 & 20 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 10 & 20 \end{vmatrix}} = \frac{5460 - 4370}{20 - 10} = 109$$

$$N = \frac{\begin{vmatrix} 1 & 273 \\ 10 & 4370 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 10 & 20 \end{vmatrix}} = \frac{4370 - 2730}{20 - 10} = \frac{1640}{10} = 164$$

Remarks: Candidates can apply any method of solving two linear equations to obtain these answers.

EXAMINER'S COMMENTS
This question was the next most popular after question four. Candidates were ill-prepared for optimization questions. Most candidate could not calculate simple cost functions such as the average cost and marginal cost. Most candidates differentiated the total cost function for the production level at which the average cost will be lowest. Also, forming the linear equations for the problem statement in the (b) part of the question was really difficult for some candidates. However, there were very good answers from high performing candidates with four of them scoring 20/20, 20/20 19/20 and 18/20 respectively

QUESTION SIX

(a)

(i) A sample is a subset of the population under study.
A population is set of all objects under consideration.

(ii) A Statistics is an estimate from the sample.
A Parameter is an estimate from the population.

(b)

(i) Sample mean of $\bar{X} = \frac{220}{11} = 2$, Variance $S_X^2 = \frac{440}{11-1} = 44$,

Standard Deviation $S_X = \sqrt{44} = 6.63$

(ii) $\bar{Y} = \frac{385}{11} = 35$, $S_Y^2 = \frac{1120}{11-1} = 112$, $S_Y = \sqrt{112} = 10.58$

(iii) Covariance $Cov(X,Y) = \frac{-568}{11-1} = -56.8$

(iv) Correlation Coefficient
$$Corr(X,Y) = \frac{-56.8}{6.63 \times 10.58} = \frac{-56.8}{70.15} = 0.8097$$

EXAMINER'S COMMENTS

Question Six was the second least popular question among candidates but most candidates who attempted this question did extremely well. However, some candidates found the computation of the covariance and the correlation very difficult. Only a handful could manage with the computation of the two measures of association. I recommend candidates master the theory part of the topics because their understanding of the applied part of questions are dependent on the theory.

QUESTION SEVEN

$P = 3000, i = 0.02, r = 0.02, t = 1$

$A = P(1 + rt)$

$A = 3000(1 + 0.02 = 3060$

Year	Loan	Interest	Annual Payment	Outstanding Balance
1	3060.0	61.2	289.4	2831.8
2	2831.8	56.6	289.4	2599.0
3	2599.0	52.0	289.4	2361.6
4	2361.6	47.2	289.4	2119.4
5	2119.4	42.4	289.4	1872.4
7	1872.4	37.4	289.4	1620.4
8	1363.5	32.4	289.4	1363.4
9	1101.4	27.3	289.4	1101.4
10	834.0	22.0	289.4	834.0
11	561.3	16.7	289.4	561.3
12	561.3	11.2	289.4	283.1
13	283.1	5.7	289.4	0

$$R = \frac{iP}{1 - (1 + i)^{-n}} = 1 - \frac{0.02(3060)}{(1 + 0.02)^{-12}} = 289.4$$

EXAMINER'S COMMENT

Answers provided by candidates for Question Seven (a) were not understandable . Candidates could not construct good amortization table. Some candidates could not distinguish between normal distributions question and business mathematics question. Candidates do not understand the principle of compound interest and deferred payment. In fact this question is the least patronized question by candidates.

Finally, the marking exercise went quite well with many examiners marking under less or no pressure during the residential marking.

CONCLUSION

Candidates notable strengths in the performance were on the forecasting question (**QUESTION   TWO**), probability (**QUESTION THREE**) and Calculus & Linear equations (**QUESTION FIVE**). Majority of candidates could do time series decomposition easily and   use the calculator very well in the **QUESTION, ONE & TWO**. These strengths were demonstrated mostly by candidates who took their paper in Accra, Kumasi and Cape Coast. This might be due to the availability of teaching and learning materials in these centers, and qualified instructors for teaching the quantitative tools in business. I suggest ICAG opens teaching centres in every regional capital and also finds qualified lecturers to teach at the centers.

Candidates main weaknesses were; lack of basic knowledge of algebra and inadequate preparation for the exams. This is reflected in their inability to distinguish between the words **SAMPLE** and   **POPULATION**, and **STATISTIC** and **PARAMETER**. This weakness is widespread and included candidates from even the three high performing centres of Accra, Kumasi and Cape Coast.